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Math Instruction That Makes Sense

By Michael Zwaagstra

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About the author

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Executive Summary

- A solid understanding of mathematics, also known as numeracy, is an important component of a well-rounded education. Unfortunately, schools are largely failing in this regard.
- Provincial curriculum guides, and the textbooks recommended by them, place a great deal of emphasis on problem solving and the conceptual understanding of mathematics.
- In addition, the math curriculum and textbooks in public schools employ highly ineffective, discovery-based instructional techniques. Students do not learn the standard algorithms for math equations, and they fail to master basic math skills.
- In order for students to receive a strong grounding in math, they need to spend more time practising math skills such as basic addition and subtraction along with the standard multiplication tables.
- There is ample research evidence showing that deliberate practise is the best way to gain mastery over a particular subject or skill.
- Mastering the standard algorithms makes it possible for students to gain a deeper understanding of more-complex mathematical problems.
- John Mighton, the founder of JUMP (Junior Undiscovered Math Prodigies), found that students needed to have math problems broken down into small steps and that each step had to be mastered before moving to the next step.
- In order to improve our system of math instruction, schools must place a much stronger emphasis on mastering basic math skills and standard algorithms. Math curriculum guides must require the learning of standard algorithms, and textbooks must contain clear, systematic instructions as to their use.

It is important for our schools that students graduate with solid math skills. Not only are they essential in the workplace, they are a necessary foundation for success in many college and university programs.

> ***...for students to receive a strong grounding in math, they need to spend more time practising math skills...***

Introduction

A solid understanding of mathematics, also known as numeracy, is an important component of a well-rounded education. The ability to perform basic mathematical computations is a requirement of many entry-level jobs. In addition, careers in fields such as engineering, medicine, finance and all of the sciences require a solid background in higher-level university mathematics, including calculus, statistics and linear algebra.

Because math is such an important skill, schools have an obligation to ensure that students learn key math concepts. Unfortunately, schools are largely failing in this regard. First-year post-secondary students are increasingly unprepared for university-level mathematics, and this has led to a proliferation of remedial math courses at universities across Canada. Many parents choose to enroll their children in special tutoring sessions with organizations such as Kumon and the Sylvan Learning Centre to fill in the gaps left by the public school system. Unfortunately, many cannot afford extra tutoring, and this creates a two-tiered system that unfairly penalizes children whose parents cannot pay for extra math lessons.

Although there is solid evidence supporting the traditional approaches to teaching math that involve mastering standard algorithms,[1] practising skills to mastery and introducing concepts in incremental steps, most provincial math curricula and textbooks employ a different approach. Constructivism, which encourages students to come up with their own understanding of the subject at hand, is the basis for this new approach to teaching math. As a result, there is very little direct instruction of important mathematics algorithms or rigorous practising and memorization of basic math facts.

Our students deserve better. Pupils who are not taught math properly are being unfairly denied the opportunity to enter careers in many desirable fields. The public school system has an obligation to ensure that every child has the opportunity to learn the mathematics required for university-level mathematics courses.

> ***First-year post-secondary students** are increasingly unprepared for university-level mathematics, and this has led to a proliferation of remedial math courses at universities across Canada.*

The weak math skills of our high school graduates

Defenders of the current system claim that Canadian students are already receiving a solid math education. They point to the Programme for International Assessment (PISA) results, which give Canada a relatively high standing compared with the rest of the world.[2/3] Out of 65 countries and economies that participated in this assessment of 15-year-old students, Canadian students came in 10th on the mathematics portion of the 2009 study.[4] With results like these, they argue, how could anyone think there is a problem with the way we teach math in our schools?

The problem with this claim is that PISA only assesses students on their understanding of "everyday math."[5] In other words, it does not evaluate the students' work with algebra, geometry, fractions or any other number of important math concepts. It is entirely possible for students to do well on the PISA exams and still be unprepared for high school math, let alone university-level math.[6] A large group of mathematicians thinks this is the case in Finland where their students do better than our students do on PISA, and yet they are not performing well at math in university.[7]

In fact, there is good reason to believe the situation is similar in Canada. University professors who are responsible for instructing first-year students work on the front lines with high school graduates. There is a strong consensus among math professors that the math skills of these students are much weaker than they were two or three decades ago.[8/9]

Pengfei Guan, a professor of mathematics at McGill University and former vice-president of the Canadian Mathematical Society, lays the blame directly at the feet of the math instruction provided in public schools. "Like many of my colleagues, I am dismayed by the state of the mathematics education in elementary and secondary schools. There is a well documented discussion of problems in mathematics education among mathematicians."[10]

Since 1990, Jo-Anne LeFevre, a psychology professor at Carleton University, has conducted research on the ability of first- and second-year students to answer simple arithmetic questions. Over the past 20 years, she has observed a 25 per cent decline in the number of questions students were able to answer correctly within the same time limit. Another psychology professor, Brenda Smith-Chant of Trent University, conducted a similar experiment and found an even sharper decline in the math skills of students at her university.[11]

Clearly, something is wrong with the state of math education in Canada. The status quo is not acceptable.

> ***Like many of my colleagues, I am dismayed by the state of the mathematics education...***
>
> ***— Pengfei Guan, professor of mathematics at McGill University***

How math is currently taught in school

Provincial curriculum guides and the textbooks recommended by them place a great deal of emphasis on problem solving and the conceptual understanding of mathematics. Accurate calculations receive considerably less emphasis. This reflects the official position of the National Council of Teachers of Mathematics, which has published standards that de-emphasize the learning of factual content and procedures.

> *Most of the arithmetic and algebraic procedures long viewed as the heart of the school mathematics curriculum can now be performed with handheld calculators. Thus, more attention can be given to understanding the number concepts and the modeling procedures used in problem solving.*[12]

The Western and Northern Canadian Protocol (WNCP) math curriculum document also reflects this de-emphasis on drill and practice.[13] WNCP establishes a common curriculum framework for the provinces of British Columbia, Alberta, Saskatchewan and Manitoba as well as the three territories. There are many references in this framework to conceptual understanding of math but virtually none to mastering the standard algorithms through drill and practice.

However, there is a big difference between demonstrating a conceptual understanding of mathematics and actually being able to solve equations accurately and efficiently. Just as most people would be very uncomfortable giving a driver's licence to someone who merely demonstrates a conceptual understanding of how to drive a car, we should be concerned about a math curriculum that fails to emphasize the importance of mastering basic math skills.

One individual whose books and articles significantly influenced the current curriculum framework is former Virginia Commonwealth University education professor John Van de Walle. Until his death in 2006, he advocated the problem-based method for teaching mathematics and strongly discouraged teachers from using standard algorithms to solve equations. Through this student-centered approach, students develop their own understanding of math and invent their own ways of answering math questions. Van de Walle also made it clear that he considered his problem-based technique to be completely incompatible with the traditional approach of teaching standard algorithms.[14]

The influence of Catherine Fosnot, an education professor at City University in New York, also comes through loud and clear. Like Van de Walle, Fosnot encourages teachers to have students invent their own math algorithms instead of teaching them standard algorithms, and she discourages teachers from using direct instruction. Underpinning this whole approach is a theory of learning known as constructivism.

> *Learning—deep, conceptual learning—is about structural shifts in cognition. It is about self-organizing at moments of criticality. These changes are complex and non-linear, and they are the result of interacting autopoietic systems.*
>
> *From a constructivist perspective, meaning is understood to be the result of humans setting up relationships, reflecting on their actions, and modeling and constructing explanations.*[15]

In other words, do not expect Fosnot or her followers to present students with the most-efficient algorithms for solving mathematics problems. They are far more interested in the dynamics in human relationships and constructing meaning than in ensuring students acquire the specific math skills they need to progress to higher levels of learning. This is a dangerous gamble and may shut out large numbers of individuals from careers in the sciences.

A specific example: Two-digit multiplication

According to the WNCP, all Grade 5 students are expected to, "[d]emonstrate an understanding of multiplication (2-digit by 2-digit) to solve problems."[16]

However, the specific outcomes listed beside the general outcome seem geared toward anything but a proper understanding of multiplication.

- Illustrate partial products in expanded notation for both factors, e.g., for 36 × 42, determine the partial products for (30 + 6) × (40 + 2).
- Represent both 2-digit factors in expanded notation to illustrate the distributive property, e.g., to determine the partial products of 36 × 42, (30 + 6) × (40 + 2) = 30 × 40 + 30 × 2 + 6 × 40 + 6 × 2 = 1200 + 60 + 240 + 12 = 1512.
- Model the steps for multiplying 2-digit factors using an array and base ten blocks, and record the process symbolically.
- Describe a solution procedure for determining the product of two given 2-digit factors using a pictorial representation, such as an area model.
- Solve a given multiplication problem in context using personal strategies and record the process.[17]

Nowhere does it emphasize that students need to answer multiplication questions accurately nor does it mandate that students learn the most efficient multiplication algorithm. Instead, the curriculum says students should model 2-digit factors with an array and base ten blocks, draw pictures of the solution procedure and use "personal strategies" to solve multiplication questions.

This inefficient and confusing form of teaching multiplication is clearly present in two of the new math textbooks commonly used in math classrooms—Pearson Education Canada's *Math Makes Sense 5*[18] and Nelson Education's *Math Focus 5*.[19]

Using the sample question 21 × 13, *Math Makes Sense* shows three different techniques for solving this question. In one technique, the problem is modelled with base ten blocks and the student adds 2 hundreds (200) + 7 tens (70) + 3 ones (3) and gets 273. Another technique involves drawing an array on grid paper with 13 rows and 21 squares in each row and then adding 200 + 60 + 10 + 3 to get 273. The third technique involves drawing a similar multiplication array and then writing each factor in expanded form with four partial products:

$$
\begin{aligned}
21 \times 13 &= (20 + 1) \times (10 + 3) \\
&= (20 \times 10) + (20 \times 3) + (1 \times 10) + (1 \times 3) \\
&= (200 + 60 + 10 + 3) \\
&= 273
\end{aligned}
$$

Students receive a series of questions to work on using these strategies. They are also encouraged to invent their own strategies to solve multiplication questions.

Math Focus recommends the same techniques as *Math Makes Sense* but manages to present them in an even more confusing manner. Here is how it recommends multiplying 23 × 11:

First, students should model an array of 23 rows of 11 squares but minimize the number of base ten blocks used by reducing 23 to 20 and 11 to 10. In the second step, they write out the products:

$$
\begin{aligned}
20 \times 10 &= 200 \\
20 \times 1 &= 20 \\
3 \times 10 &= 30 \\
3 \times 1 &= 3
\end{aligned}
$$

Then they add the four products to get the final answer.

$$
\begin{array}{rl}
11 & \\
\times\ 23 & \\
\hline
200 & (20 \times 10) \\
20 & (20 \times 1) \\
30 & (3 \times 10) \\
+3 & (3 \times 1) \\
\hline
253 &
\end{array}
$$

Math Focus manages to take what should be a relatively simple math question (11 × 23) and turns it into an incredibly complicated and confusing one. Imagine trying to use this or one of the other recommended techniques to multiply large three-digit numbers! This is probably why the curriculum explicitly states that students should use a calculator when multiplying numbers with more than two digits.[20] No wonder our students have such poor math skills when they enter university.

In contrast, older math textbooks show students the traditional algorithm for multiplying two-digit numbers. Ginn and Company's *Starting Points in Mathematics 5*[21] tells students to write 36 under 42 and then multiply one digit at a time. After multiplying 6 × 42, students write a zero on the next line to hold the tens spot, multiply 3 × 42 and add the two products together.

$$
\begin{array}{r}
42 \\
\text{x } 36 \\
\hline
252 \\
+1260 \\
\hline
1512
\end{array}
$$

Not only is this traditional algorithm accurate and more efficient than the ones promoted in the new textbooks, it has the added bonus of being easily adapted to work with larger numbers. Another 30-year old textbook, Copp Clark Pitman's *Mathways 5*[22] also shows students the standard multiplication algorithm. Unlike the new textbooks, *Mathways* and *Starting Points in Mathematics* keep their directions simple and do not confuse students with techniques that only work well with smaller numbers.

Although some may claim the standard algorithm does not promote conceptual understanding, it should be easy for a knowledgeable math teacher to use the distributive property to demonstrate why this algorithm works. Furthermore, this algorithm can be employed efficiently with numbers of any size. Writing out questions in this vertical fashion rather than in the much longer format suggested in the new textbooks saves time and minimizes confusion. Thus, when taught correctly, mastering the standard multiplication algorithm promotes both understanding and competence.

Back to the basics: The way forward

In order for students to receive a strong grounding in math, they need to spend more time practising math skills such as basic addition and subtraction along with the standard multiplication tables. Although the current curriculum says students should have an understanding of basic multiplication, this does not go far enough. Students must practise their basic math facts frequently for it to become automatic. There is ample research evidence showing that deliberate practise is the best way to gain mastery over a particular subject or skill.[23]

In addition, the math curriculum needs to reflect the importance of standard algorithms. W. Stephen Wilson, a professor of mathematics at Johns Hopkins University, aptly summarizes why standard algorithms must be a part of math instruction:

> *Students must study arithmetic. The standard algorithms for whole numbers are the only really big theorems that students can be taught in elementary school. It is deep, beautiful, and powerful mathematics. Master these algorithms with understanding, and you're ready to go.*[24]

It is common to hear math educators and curriculum consultants claim that a conceptual understanding of mathematics and the traditional emphasis on basic skills and standard algorithms are mutually exclusive.[25] However, this is a false dichotomy. Mastering the standard algorithms makes it possible for students to gain a deeper understanding of more-complex mathematical problems. In addition, when students learn how standard algorithms work, they are simultaneously learning the algorithms and gaining a conceptual underpinning.[26]

Another advantage of learning standard algorithms and basic math facts is that this frees up space in the short-term memory for more advanced tasks. Cognitive psychologist Daniel Willingham points out that there is a limited amount of available space in our working memory, and when skills become automatic, they no longer take up the same amount of room as they did before.[27] For example, after a lot of practice, basic number facts such as simple addition and multiplication tables become automatic. This then frees up space in short-term memory for more advanced math tasks. In contrast, the student who struggles to figure out 6×4 will quickly become lost in solving an algebra problem such as $(6x + 5)(4x) = 20$.

John Mighton, the founder of JUMP (Junior Undiscovered Math Prodigies), discovered this principle when tutoring students who had weak math skills. He found that students needed to have math problems broken down into small steps and that each step had to be mastered before moving to the next step. Although this technique, often referred to as scaffolding, went against the problem-based approach to teaching math currently employed in public schools, it proved to be highly effective.[28]

In a randomized control study involving more than 300 Canadian fifth grade students, those who received instruction in the JUMP program were compared with those who were taught the regular curriculum. Students in the JUMP program achieved more than double the growth in mathematical competencies after only five months.[29] There are many other success stories from this program, and its popularity is growing every year.[30/31]

Students need to master the basics in order to succeed in math. For the majority of them, this means, for example, the sequential teaching of standard math algorithms and practising basic number facts to mastery. Just as someone who does not practise the piano will never learn to play well, someone who does not practise basic math skills will never become fluent in math.

> ***Students in the JUMP program achieved more than double the growth in mathematical competencies after only five months.***

Conclusion

It is important for our schools that students graduate with solid math skills. Not only are they essential in the workplace, they are a necessary foundation for success in many college and university programs. We cannot afford for our students to fall behind those in other countries.

Unfortunately, the math curriculum and textbooks in public schools employ highly ineffective, discovery-based instructional techniques. Students do not learn the standard algorithms for math equations, and they fail to master basic math skills. This inefficient way of teaching math does not serve our students well.

In order to improve our system of math instruction, schools must place a much stronger emphasis on mastering basic math skills and standard algorithms. Math curriculum guides must require the learning of standard algorithms, and textbooks must contain clear, systematic instructions as to their use. As the success of John Mighton's JUMP program illustrates, all students are capable of mastering math if they receive the best instruction.

Endnotes

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Further Reading

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Dennis Owens

http://www.fcpp.org/publication.php/804

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